Limited carry-in technique for real-time multi-core scheduling

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ABSTRACT

Schedulability analysis has been widely studied to provide offline timing guarantees for a set of real-time tasks. The so-called limited carry-in technique, which can be orthogonally incorporated into many different multi-core schedulability analysis methods, was originally introduced for Earliest Deadline First (EDF) scheduling to derive a tighter bound on the amount of interference of carry-in jobs at the expense of investigating a pseudo-polynomial number of intervals. This technique has been later adapted for Fixed-Priority (FP) scheduling to obtain the carry-in bound efficiently by examining only one interval, leading to a significant improvement in multi-core schedulability analysis. However, such a successful result has not yet been transferred to any other non-FP scheduling algorithms. Motivated by this, this paper presents a generic limited carry-in technique that is applicable to any work-conserving algorithms. Specifically, this paper derives a carry-in bound in an algorithm-independent manner and demonstrates how to apply the bound to existing non-FP schedulability analysis methods for better schedulability.

1. Introduction

Schedulability analysis determines whether a set of real-time tasks can meet any given timing constraints (i.e., deadlines) under a certain scheduling algorithm on a specific computing platform. A substantial number of studies have been made to analyze schedulability in multi-core scheduling, introducing some successful results such as optimal scheduling algorithms for certain types of tasks (e.g., periodic tasks with implicit-deadlines). However, it yet lacks full understanding of how tasks behave on multi-cores under a given algorithm, particularly, for more general task types (e.g., periodic/sporadic tasks with constrained-deadlines). This leads to the development of many sufficient (but not exact) schedulability analysis methods [1–3].

Many different analysis methods share the perspective of investigating interference – how long the execution of a job of interest can be delayed due to the execution of other higher-priority jobs. Different analysis methods differ in how to derive a tight bound on the amount of such interference delay. Critical to this is how to tightly compute the contribution of carry-in jobs to the interference. A job is said to be a carry-in job for a given interval, if the job is released before the interval and does not yet finish its execution at the beginning of the interval.

A few studies have been made to compute the contribution of carry-in jobs. Baruah [2] introduced the so-called limited carry-in technique for Earliest Deadline First (EDF) [4] utilizing the concept of busy intervals. This technique requires to examine a pseudo-polynomial number of intervals for a tight carry-in bound. Guan et al. suggested to obtain a carry-in bound efficiently by examining only one interval under Fixed-Priority (FP) scheduling [4], taking advantage of the critical instant of a job (that maximizes its response time) under FP scheduling [5]. This technique yields reducing the carry-in bound substantially, resulting in a significant improvement in schedulability analysis for FP. A recent study [6] showed that such an FP-specific limited carry-in technique is applicable to a subset of tasks even under a dynamic-priority scheduling algorithm called Smallest Pseudo-Deadline First (SPDF), when the subset of tasks exhibits FP-oriented priority relationships between each other.

However, such an effective (in schedulability improvement) and efficient (in time-complexity) limited carry-in technique has not been utilized for other non-FP scheduling algorithms. Therefore, this paper seeks to develop a generic limited carry-in technique that can be applied to any work-conserving algorithms, under which any core is not idle as long as there is an unfinished job. To this end, we first recapitulate existing schedulability analysis techniques (in Section 2). Then, we develop a new carry-in bound, which can be applied to any work-conserving algorithm (in Section 2). Finally, we show applications how the new bound can be incorporated into existing schedulability analyses for better schedulability (in Section 3).

System Model. In this paper, we focus on a sporadic task model [7], in which a task τ i in a task set τ is specified by the minimum separation T i, the worst-case execution time C i, and the relative
We focus on constrained deadline tasks, i.e., $C_i \leq D_i \leq t_i$. A task $t_i$ invokes a series of jobs, each separated from its predecessor by at least $t_i$ time units, and supposed to finish its execution with $D_i$ time units. Each job cannot be executed in parallel.

Also, we consider a multi-core system consisting of $m$ identical cores, and preemptive work-conserving global algorithms, under which a higher-priority job can preempt a lower-priority job any time (preemptive), any core cannot be idle if there is an unfinished job (work-conserving), and a job can migrate from one core to another (global). Without loss of generality, let one time unit denote the quantum length, and all task parameters are assumed to be specified as multiples of the quantum length.

2. Limited carry-in technique for any work-conserving algorithm

Many schedulability analysis methods employ the concept of interference. Let $I_{k,a}(t,b)$ denote the interference of $t_k$ on $t_a$ in $[a,b]$, meaning the cumulative length of all intervals in $[a,b]$ such that a job of $t_a$ executes but the job of $t_k$ of interest cannot although it is ready to execute.

Then, considering that a job of $t_k$ cannot be executed in a time slot, only when $m$ other jobs are executed, the interference-based schedulability analysis framework has been developed in [1,3], as follows.

**Lemma 1.** (Theorem 3 in [1], Theorem 6 in [3]) Every job of a task $t_k \in \tau$ finishes its execution within $t$ time units if the following inequality holds for $t \leq D_k$:

$$C_k + \frac{1}{m} \max_{t \leq \ell} \sum_{t_1 \leq t_2 \leq \ell} \min\{I_{k,a}(t,t+\ell),\ell-C_0+1\} \leq \ell,$$  

where $T \triangleq \{t \text{ the release time of a job of } t_k \}$. Then, if Eq. (1) holds for all $t_1 \in \tau$, we deem $\tau$ is schedulable.

It is difficult to calculate the exact value of $I_{k,a}(t,t+\ell)$ offline, because the value depends not only on the scheduling algorithm, but also on job release patterns before $t$. Hence, existing schedulability analysis techniques have derived two types of upper-bounds on $I_{k,a}(t,t+\ell)$: (i) algorithm-independent ones and (ii) algorithm-specific ones.

Regarding (i), a release (and execution) pattern that maximizes the execution of jobs of $t_i$ has been identified [1]. As shown in Fig. 1(a), this pattern allows a task $t_i$ to have a carry-in job in the interval of interest. In the figure, the first (carry-in) job executes as late as possible, and the following jobs execute as early as possible. Here $S_i$ means the slack value of $t_i$, a difference between the finishing time and deadline of a job of $t_i$; it has been detailed in [1,3] how to calculate $S_i$ with existing schedulability analyses. Then, the amount of the maximum execution of jobs of $t_i$ in an interval of length $\ell$ is calculated by [1,3]

$$W^{CI}(\ell) + N^{SI}(\ell) \cdot C_i + \min\{C_i,\ell + D_i - S_i - C_i - N^{SI}(\ell) \cdot T_i\},$$  

where $N^{SI}(\ell)$ denotes the number of jobs of $t_i$ whose deadlines are within an interval of length $\ell$ (e.g., the first two jobs in Fig. 1(a)), and calculated by

$$\frac{(\ell - S_i - C_i)}{C_0}\frac{(\ell - C_i)}{C_0}.$$  

Since a job can interfere with another job only when it is executed, $I_{k,a}(t,t+\ell) \leq W^{CI}(\ell)$ holds for all $t \neq \ell$ and $t_i \in \tau$. Theorem (1) with replacing $I_{k,a}(t,t+\ell)$ by $W^{CI}(\ell)$ yields a safe schedulability analysis of any work-conserving algorithm [1,3], and this schedulability analysis assumes that all tasks can have carry-in jobs.

In case that a task $t_i$ cannot have any carry-in job in an interval of length $\ell$, Fig. 1(b) illustrates a release (and execution) pattern that maximizes the execution of jobs of $t_i$. Here the number of jobs of $t_i$ whose release time and deadline are in the interval is $N^{HC}(\ell) \triangleq \lfloor \ell/r \rfloor$, and the amount of the maximum execution of jobs of $t_i$ is calculated by [8,5]

$$W^{HC}(\ell) + N^{HC}(\ell) \cdot C_i + \min\{C_i,\ell - N^{HC}(\ell) \cdot T_i\}.$$  

We can easily check $W^{HC}(\ell) = W^{CI}(\ell)$ for all $\ell \geq 0$ and $t_i \in \tau$.

Since we do not know offline whether a task has its carry-in job in the interval of interest, $W^{HC}(\ell)$ cannot be a safe upper-bound of $I_{k,a}(t,t+\ell)$. Instead, $I_{k,a}(t,t+\ell) \leq W^{HC}(\ell)$ holds only if $t_i$ does not have any carry-in job in $[t,t+\ell]$. However, we can derive a safe upper-bound on the amount of execution with limited carry-in jobs, as stated in the following lemma.

**Lemma 2.** Let $\Gamma$ denote a set of intervals (not necessarily continuous) of length $\ell - x (x \geq 0)$ over $[t,t+\ell]$. Suppose there are at most $m - 1$ tasks which have their carry-in jobs in $[t,t+\ell]$. Then, the amount of execution of jobs of tasks in $\tau$ in any $\Gamma$ is upper-bounded by $\mathcal{F}(\ell,x)$, where

$$\mathcal{F}(\ell,x) \triangleq \min\{W^{HC}(\ell),\ell-x\} + \sum_{m-1 \text{ largest } t_i \in \tau} \min\{W^{CI}(\ell),\ell-x\} - \min\{W^{HC}(\ell),\ell-x\}.$$  

**Proof.** Recall that $W^{CI}(\ell)$ and $W^{HC}(\ell)$ represent the maximum amount of execution of jobs of $t_i$ in an interval of length $\ell$, respectively when there is a carry-in job and no carry-in job of $t_i$ in the interval. Considering the length of $\Gamma$ is $\ell - x$, $\min\{W^{CI}(\ell),\ell-x\}$ (likewise $\min\{W^{HC}(\ell),\ell-x\}$) is an upper-bound on the maximum amount of execution of jobs of $t_i$ in $\Gamma$ in case of the existence of a carry-in job of $t_i$ (likewise no carry-in job of $t_i$).

Since we do not know which task has its carry-in job, we initially add $\min\{W^{HC}(\ell),\ell-x\}$ for all tasks in $\tau$. Then, we add the difference between $\min\{W^{CI}(\ell),\ell-x\}$ and $\min\{W^{HC}(\ell),\ell-x\}$ for $m-1$ tasks with the largest difference. Then, for any combination of at most $m-1$ tasks with carry-in jobs, $\mathcal{F}(\ell,x)$ in Eq. (4) is a safe upper-bound on the amount of execution of jobs of tasks $t_i$ in any $\Gamma$, provided that there are at most $m-1$ tasks with carry-in jobs in $[t,t+\ell]$. □

Using the lemma, we finally present the main theorem of this paper.
Theorem 1. Under any work-conserving algorithm, the total amount of interference in Eq. (1) is upper-bounded as follows:

\[
\max_{i \in \mathcal{T}} \sum_{\tau_i \in \mathcal{T} - \{\tau_i\}} \min \{I_{k-i}(t, t + \ell), \ell - C_k + 1\} \leq \mathcal{F}(\ell, C_k - 1) \text{ in Eq. (4).}
\] (5)

Proof. Suppose that Eq. (1) with replacing the LHS of Eq. (5) by the RHS satisfies the following inequality for \( \ell \leq D_w \):

\[
C_k + \left\lfloor \frac{1}{m} \left( \mathcal{F}(\ell, C_k - 1) \text{ in Eq. (1)} \right) \right\rfloor \leq \ell
\]

\[\iff \mathcal{F}(\ell, C_k - 1) \text{ in Eq. (4)} < m \cdot (\ell - C_k + 1). \] (6)

Then, we will prove that any job of \( \tau_i \) finishes its execution within \( \ell \) time units. Without loss of generality, we focus on a job of \( \tau_i \) whose release time and deadline are \( t_0 \) and \( t_0 + D_k \).

We choose a time instant \( t_0 - x \) \((x > 0)\) such that an interval \([t_0 - x, t_0]\) is the maximum busy interval, in which all m cores are occupied in \([t_0 - x, t_0]\), but at least one core is idle in \([t_0 - 1, t_0 - x]\). Then, by definition, there are at most \( m - 1 \) tasks which have their carry-in jobs in an interval starting from \( t_0 - x \).

We consider two cases: (i) \( 0 < x < \ell - C_k + 1 \) and (ii) \( x \geq \ell - C_k + 1 \).

In case of (i), we focus on \([t_0 - x, t_0] \cup \Gamma \) where \( \Gamma \) denotes a set of intervals (not necessarily continuous) of length \( \ell - C_k + 1 - x \) over \([t_0 - x, t_0 + \ell]\); \( \Gamma \) is chosen such that the amount of execution of jobs of tasks in \( \Gamma \) is maximized. If we apply Lemma 2 with \( t = t_0 - x \) and \( \mathcal{T} = [t_0 - x, t_0) \cup \Gamma \), Lemma 2 and Eq. (6) imply that the amount of execution of jobs of tasks in \( \Gamma \) in \([t_0 - x, t_0 - x + \ell]\) (because the interval is a busy interval), there are strictly less than \( m \cdot \ell - (C_k + 1) \) executions in \( \Gamma \), which means at least one time slot in \( \Gamma \) is a non-busy slot in which at least one core is idle. Then, by the definition of \( \Gamma \), each time slot in \([t_0 - x, t_0 - x + \ell]\) \( \Gamma \) is non-busy. Therefore, the job of interest can be executed in all time slots in \([t_0, t_0 - x + \ell]\) \( \Gamma \) (whose length is \( C_k - 1 \)) and at least one slot in \( \Gamma \), which means the job of interest finishes \( C_k \) amount of execution in \([t_0, t_0 - x + \ell]\).

In case of (ii), we focus on \([t_0 - x, t_0 - x + \ell - C_k + 1]\). We apply Lemma 2 with \( t = t_0 - x \) and \( \mathcal{T} = [t_0 - x, t_0 - x + \ell - C_k + 1]\) where \( \Gamma \) denotes a set of intervals (not necessarily continuous) of length \( \ell - C_k + 1 - x \) over \([t_0 - x, t_0 + \ell]\) is strictly less than \( m \cdot \ell - (C_k + 1) \). By the definition of \( \mathcal{T} \), each time slot in \([t_0 - x, t_0 - x + \ell - C_k + 1]\) is a busy interval, and therefore the amount should be exactly as much as \( m \cdot (C_k - 1) \). Therefore, if Eq. (6) holds, \([t_0 - x - x + \ell - C_k + 1]\) cannot be a busy interval, which is a contradiction.

3. Applications of the limited carry-in technique

While Theorem 1 can be combined with most (if not all) interference-based schedulability analyses, we now present examples of the application.

In [3], a schedulability analysis for any work-conserving algorithm has been developed, by using \( W_i^C(\ell) \) as an upper-bound of \( I_{k-i}(t, t + \ell) \) as follows:

\[
\max_{i \in \mathcal{T}} \sum_{\tau_i \in \mathcal{T} - \{\tau_i\}} \min \{I_{k-i}(t, t + \ell), \ell - C_k + 1\} \leq \sum_{\tau_i \in \mathcal{T} - \{\tau_i\}} \min \{W_i^C(\ell), \ell - C_k + 1\}. \] (7)

In [1], a schedulability analysis for a given scheduling algorithm has been developed, by deriving an algorithm-specific upper-bound on \( I_{k-i}(t, t + \ell) \) as follows:

\[
\max_{i \in \mathcal{T}} \sum_{\tau_i \in \mathcal{T} - \{\tau_i\}} \min \{I_{k-i}(t, t + \ell), \ell - C_k + 1\} \leq \sum_{\tau_i \in \mathcal{T} - \{\tau_i\}} \min \{W_i^{C_a}(\ell), E_{k-i}, \ell - C_k + 1\}. \] (8)

where \( E_{k-i} \) is an algorithm-specific upper-bound on \( I_{k-i}(t, t + \ell) \) for any \( \ell \leq D_k \). For example, under EDF and Earliest Deadline first until Zero-latixy (EDZL) [9], \( E_{k-i} \) is calculated by \( n_i^E(D_k - C_k + 1) \), \( C_k + \max(0, D_k - N_i^E(D_k - T_k - S_k)) \) \[1,10\], considering a job with later deadline cannot interfere with another job with earlier deadline under EDF, and the former can interfere with the latter only when the former enters the zero-latixy state under EDZL.

If we apply Theorem 1, we can derive tighter upper-bounds than the ones presented in Eqs. (7) and (8). For the schedulability analysis for any work-conserving algorithm, we use the following upper-bound:

\[
\text{LHS of Eq. (7)} \leq \min \{\text{RHS of Eq. (7)}, \text{RHS of Eq. (5)}\}. \] (9)

Similarly, for the schedulability analysis for a given scheduling algorithm, we use the following upper-bound:

\[
\text{LHS of Eq. (8)} \leq \min \{\text{RHS of Eq. (8)}, \text{RHS of Eq. (5)}\}. \] (10)

Here we present two task sets which are not schedulable with the upper-bound in the RHS of Eqs. (7) or (8), but schedulable with the upper-bound in the RHS of Eqs. (9) or (10).

Example 1. Consider a set of three tasks \( \tau = \{\tau_T(t_1 = 4, C_1 = 1), D_1 = 4\}, \tau_2(t_2 = 4, 2, 4), \tau_2(t_2 = 4, 2, 4) \) on a two-core platform, and its schedulability is tested by the state-of-the-art schedulability analysis framework, i.e. response-time analysis [1]. Using the RHS of Eq. (7), we cannot guarantee that \( \tau_1 \) finishes its execution within 4 time units under any work-conserving algorithm. However, with the RHS of Eq. (9), we can guarantee that all three tasks meet their deadlines under any work-conserving algorithm.

Example 2. Consider a set of four tasks \( \tau = \{\tau_1(t_1 = 2, C_1 = 1), D_1 = 1\}, \tau_2(t_2 = 1, 2, 2), \tau_2(t_2 = 1, 2, 2) \) \( \tau_2(t_2 = 1, 2, 2) \) on a two-core platform, with the same schedulability analysis framework as Example 1. Using the RHS of Eq. (8), we cannot guarantee that \( \tau_1 \) and \( \tau_2 \) finish their execution within 2 time units under EDF [1]; note that the limited carry-in technique in [2] also cannot guarantee the schedulability of \( \tau \). However, with the RHS of Eq. (10), we can guarantee that all four tasks do not miss their deadlines under EDF. The same holds for EDZL; the state-of-the-art EDZL test in [10] cannot guarantee that \( \tau \) is schedulable, but we can do with the RHS of Eq. (10).

As shown in the examples, Theorem 1, once incorporated into existing schedulability analyses (e.g., Eqs. (9)), covers additional schedulable task sets which are not deemed schedulable by the existing ones.

4. Conclusion

In this paper, we derived a new upper-bound on the amount of execution under any work-conserving algorithm by limiting carry-in jobs, and demonstrated how to utilize this upper-bound to improve existing schedulability analyses. Extending the idea of the limited carry-in, it would be interesting to derive tighter,

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1. Due to the necessary condition for jobs to enter the zero-latixy state, \( E_{k-i} \) is also an upper-bound under EDZL; details are given in [10].
algorithm-specific upper-bounds. In particular, since Theorem 1 can be applied even to work-conserving non-preemptive scheduling, we expect to improve the state-of-the-art schedulability analyses for non-preemptive scheduling [11–14]. Another direction of future work is to explore other properties of our limited carry-in techniques (e.g., sustainability [15]) and extend the technique towards other systems (e.g., time-triggered embedded systems [16]).

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References


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